UNIT V

NOISE

1. Thermal noise:

- The noise generated in the resistance due to random motion of electrons, is called as thermal noise.

- Electrons in conductor possess kinetic energy due to heat exchange between conductor and its surroundings.

- Hence, electron density throughout the conductor varies randomly, thus producing random electrical voltage across the conductor.

- The relationship between noise bandwidth and rms noise voltage generated due to thermal noise is,
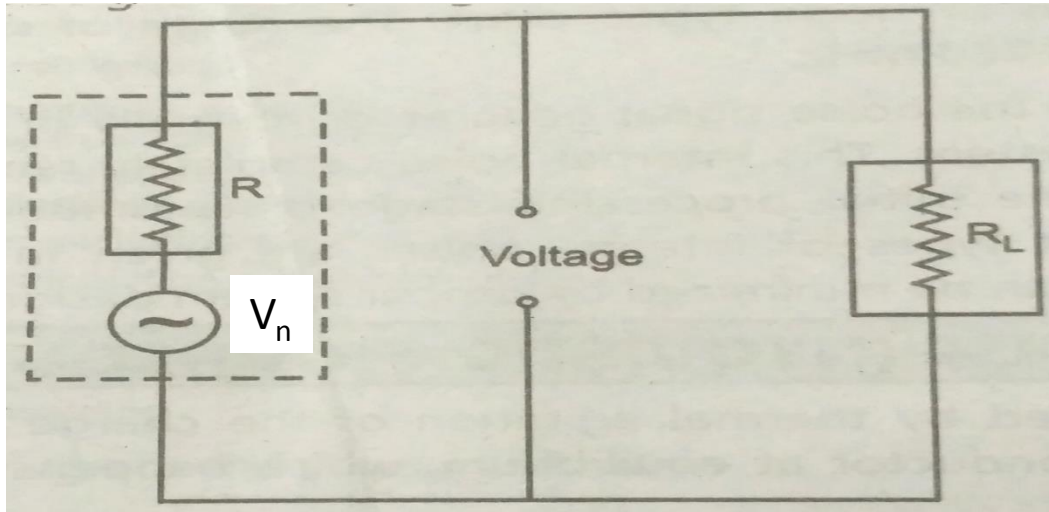
$$V_n = \sqrt{4 \times R \times K \times T \times B_n}$$

where,   R = conductor resistance

K = Boltzmann's const. = 1.38×10^{-23} J/K

T = Temperature of conductor in kelvin

B_n = Noise bandwidth in Hz

Voltage equivalent circuit of thermal noise generator

- Consider, an equivalent circuit of conductor with resistance R as noise generator as above,
- The maximum noise power output of a resistor is given as,

$$P_n = KTB_n = \text{(voltage across load)}^2/\text{(load resistance)}$$

$$= (V_n/2)^2/R_L$$

$$= (V_n/2)^2/R \quad \text{.......} \quad \text{As } R_L = R \text{ for maximum pwr transfer}$$

$$KTB_n = (V_n)^2/4R$$

Hence,

$$V_n{}^2 = 4RKTB_n$$

$$V_n = \sqrt{4 \times R \times K \times T \times B_n}$$

2. Shot noise:

Fluctuations in no. of electrons and holes under d.c. conditions constitutes the shot noise.

- Normally, it is assumed that current in an electronic device is const. under d.c. cond^n at every instant of time.
- Actually, current consists of stream of electrons and holes.
- The time average flow of electrons and holes is const. and not the actual flow of electrons and holes in the stream.
- Even though shot noise is very small as compared to the d.c. value, shot noise does contribute significantly in ampl^r circuits.
- The shot noise is given by,

$$I_n^2 = 2\ I_{dc}\ q_e\ B_n\ ampere^2$$

where, I_{dc} = direct current in amperes.

q_e = electron charge = 1.6×10^{-19} C

B_n = Noise bandwidth in Hz

3. Partition noise:

- Partition noise occurs due to random fluctuations in the division of current between two or more electrodes.

- The diode is less noisy than a transistor, as transistor has three electrodes.

- Thus, for the input stages of $\mu wave$ receivers, diode circuits are preferred.

- But, recently GaAs FET have been developed for low noise $\mu wave$ amplification, because GaAs FET has zero gate current.

4. Low frequency / Flicker noise:

- Noise whose spectral density increases with decrease in frequency below few kilohertz of frequency, is called as flicker noise (1/f noise).

- In semiconductors, flicker noise arises from fluctuations in carrier densities which give rise to fluctuations in conductivity of material.

- A noise voltage will be developed, whenever direct current flows through semiconductor and the mean-square voltage will be proportional to the square of the direct current.

- Although flicker noise is a low freq. effect, it plays an important part in limiting the sensitivity of microwave diode mixers used for Doppler radar systems. This is because, even the input frequency to the mixer are in microwave range, the Doppler frequency output is in low frequency range, where flicker noise is significant.

5. High frequency / Transit time noise:

- In semiconductor devices, the time taken by the carriers to cross a junction, is called as transit time of carriers.

- When transit time of carriers in a device is equal to period of the signal, it gives rise to transit time noise.

- When the signal frequency is high, periodic time becomes small. Thus, some of the carriers diffuse back to the source, hence, conductance of input admittance increases, which is associated with a noise current generator as

$$I_n^2 = 4GKTB_n$$

- Since, the conductance increases with the frequency, the noise spectrum density increases at high frequency.

6. Burst noise:

- The burst noise appears as a series of bursts at two or more levels. It appears in bipolar transistors and is of low frequency nature.

- The burst noise produces popping sounds in an audio systems. Hence, it's also called as "popcorn noise".

- The spectral density of burst noise increases as the frequency decreases.

7. Avalanche noise:

- Generally, the reverse bias characteristics of a diode has a very small reverse current. This reverse current may increase extremely rapidly with a slight increase in reverse-bias voltage. This region of rapid increase in reverse current is called avalanche region.

- As electrons and holes gain sufficient energy from slightly increased reverse-bias field to ionise atoms by collision, avalanche region comes in picture.

- The collisions that results in avalanche region, occurs at random, which results in large noise spikes in avalanche current.

- In zener diodes (used as vtg. ref. src.), the avalanche noise is nuisance to be avoided.

- The spectral density of avalanche noise is flat.
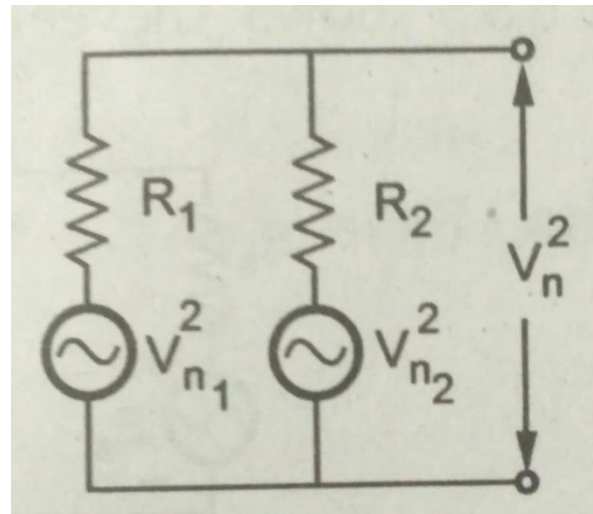
Noise calculations:

1. Series connection:

$$V_n^2 = 4\,R_s\,KTB_n$$
$$= 4\,(R_1 + R_2)\,KTB_n$$
$$= 4\,R_1\,KTB_n + 4\,R_2\,KTB_n$$
$$= V_1^2 + V_2^2$$
$$V_n = \sqrt{V_1^2 + V_2^2}$$
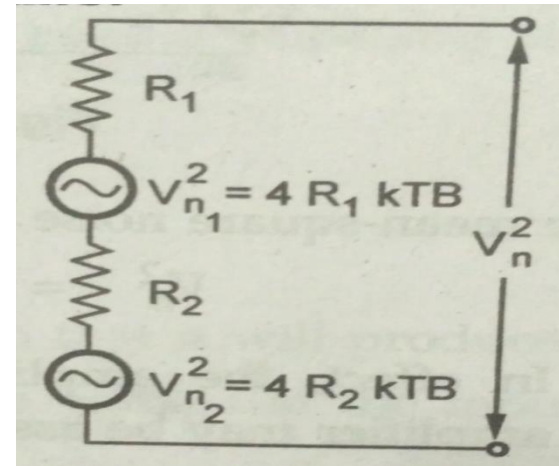
2. Parallel connection:

$$V_n^2 = 4\,R_p\,KTB_n$$
$$= 4\,(R_1 \| R_2)\,KTB_n$$
$$= 4 \cdot \frac{R_1 R_2}{R_1 + R_2} \cdot KTB_n$$

Numericals: i, iii, iv

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Equivalent noise resistance:

- The noise originating in a device (either active or passive) is represented by a fictitious resistance "R_{eq}", called as **Equivalent noise resistance**.
- This is assumed to generate noise at room temp. and then the actual device is assumed to be noiseless.

- Consider an amplifier with noise resistance "R_{eq}" and actual input resistance "R_i".
- The amplifier is driven by a signal voltage "V_s" with internal resistance "R_s" as shown below
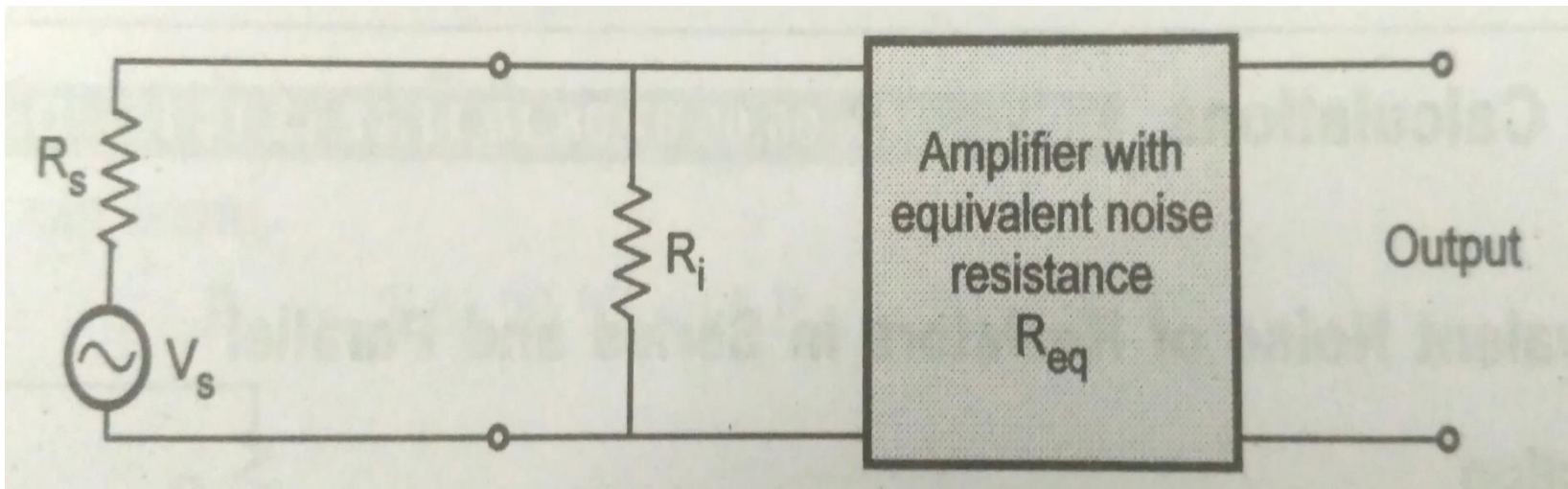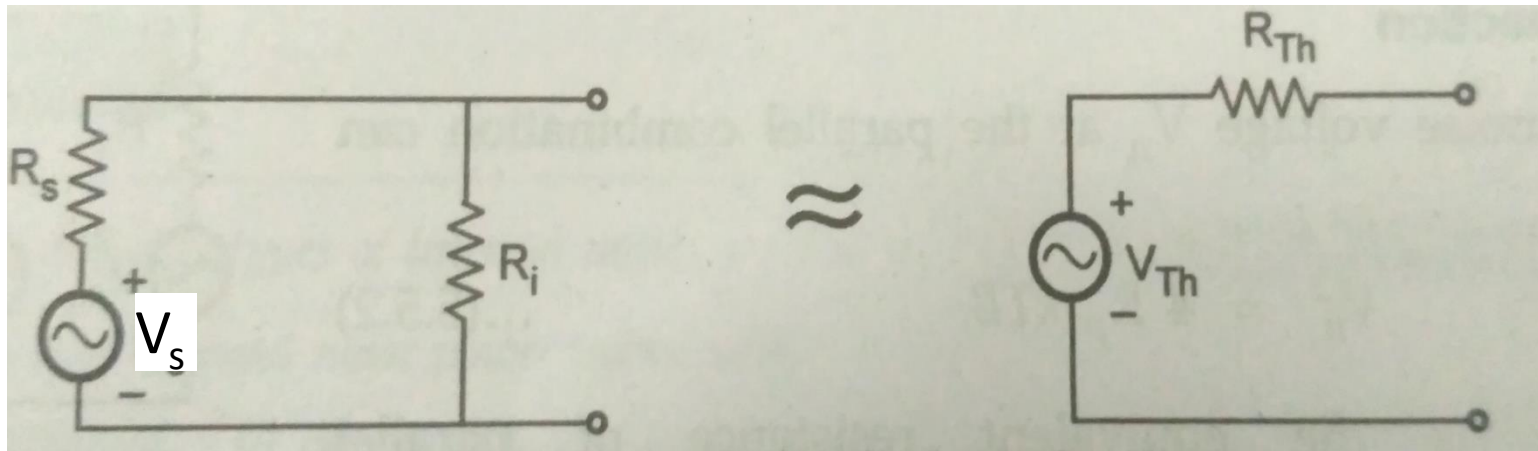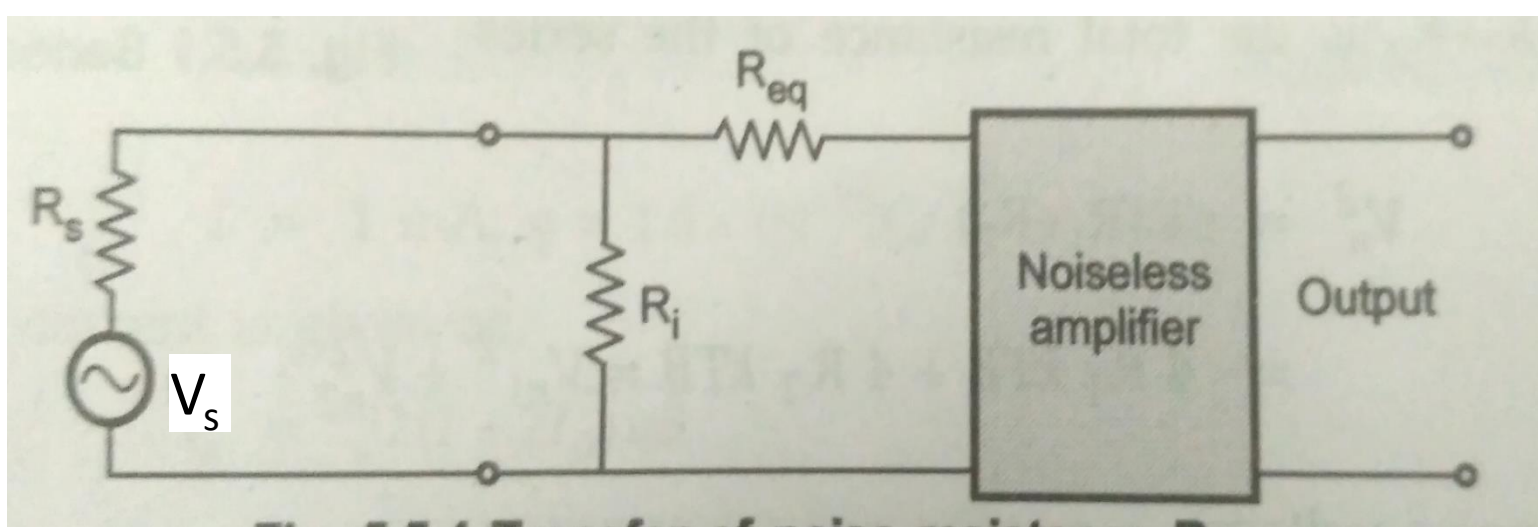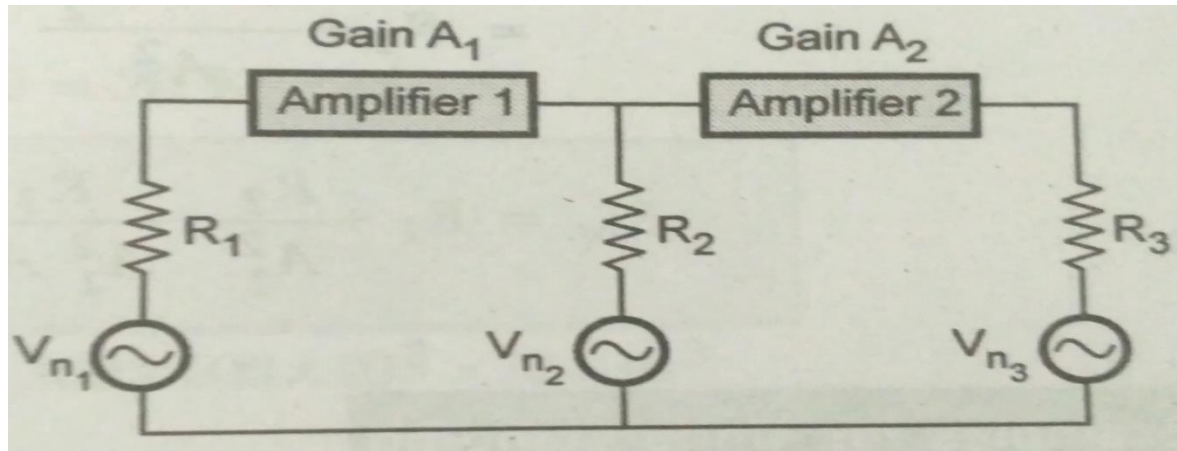
- $V_{Th} = \dfrac{V_s}{R_s + Ri} R_i$ and $R_{Th} = (R_s \parallel R_i)$

- Noise vtg. at the i/p of amplifier is
$$V_n^2 = 4 (R_{Th} + R_{eq}) KTB_n$$

Noise due to several amplifiers in cascade:

(Equivalent resistance "R_{eq}" when several amplifiers connected in cascade)

$$V_{n3} = \sqrt{4\ R_3\ KTBn}$$

If we will place R_3' at the input of "Amplr 2", so as to produce the same output as that produced with R_3. Then,

$$V_{n3}' = \frac{V_{n3}}{A_2} = \frac{\sqrt{4\ R_3\ KTBn}}{A_2} = \sqrt{\frac{4\ R_3\ KTBn}{A_2^2}} = \sqrt{4R_3'KTB_n}$$

$$R_3' = \frac{R_3}{A_2^2}$$

Thus, eq. resistance at i/p of amplr2 is,

$$R_{eq}' = R_3' + R_2 = R_2 + \frac{R_3}{A_2^2}$$

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Similarly, If we will place R_{eq}'' at the input of "Amplr 1", so as to produce the same output as that produced with R_{eq}'.
Then,

$$R_{eq}'' = \frac{R_{eq}'}{A_1{}^2}$$

$$= (R_2 + \frac{R_3}{A_2{}^2}) / A_1{}^2$$

Thus, eq. resistance at i/p of amplr1 is,

$$R_{eq} = R_1 + R_{eq}''$$

$$\boxed{R_{eq} = R_1 + \frac{R_2}{A_1{}^2} + \frac{R_3}{A_1{}^2 A_2{}^2}}$$

Numericals: ii

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Signal to noise ratio (SNR):

The ratio of signal power to noise power is called as signal to noise ratio.

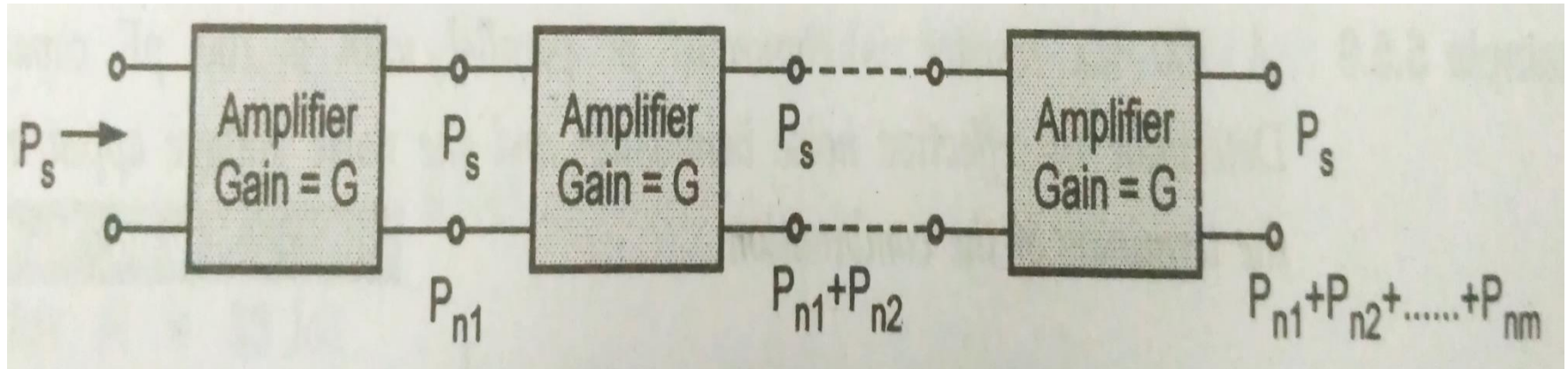$$\text{SNR} = \frac{signal\ power}{noise\ power} = \frac{P_s}{P_n}$$

$$\text{SNR in dB} = \left(\frac{S}{N}\right)_{dB} = \left(\frac{P_s}{P_n}\right)_{dB} = 10 \log \left(\frac{P_s}{P_n}\right)$$

$$= 10 \log \left[\frac{V_s^2 / R}{V_n^2 / R}\right]$$

$$\left(\frac{S}{N}\right)_{dB} = 20 \log \left(\frac{V_s}{V_n}\right) \dots.\ \text{After normalization}$$

SNR of a tandem connection:

Between the two repeaters, cable loss is L and compensation gain of amplr is G, so that LG = 1 i.e. L = 1/G   or   G = 1/L

Thus, if the input power is P_s, then power at the output of last amplr will also be P_s.

But, the noise powers are additive in nature.

$$P_{n1}+P_{n2}+P_{n3}+ \ldots \ldots + P_{nM}$$

So, Noise power at M^{th} link will be,

$$P_{n1}+P_{n2}+P_{n3}+ \ldots \ldots + P_{nM} = M \cdot P_n$$

$$\ldots \ (\text{ if } P_{n1}=P_{n2}=P_{n3}= \ldots \ldots = P_{nM} = P_n \ )$$

Hence, SNR of a tandem connection will be,

$$\left(\frac{S}{N}\right)_{\text{o/p in dB}} = 10 \log \left(\frac{P_s}{MP_n}\right)$$

$$= 10 \log P_s - 10 \log M - 10 \log P_n$$

$$= 10 \log \left(\frac{P_s}{P_n}\right) - 10 \log M$$

$$= \left(\frac{S}{N}\right)_{\text{any one link in dB}} - M_{(\text{dB})}$$

# Numericals: vi,vii

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Effect of amplification on signal to noise ratio:

The equivalent noise resistance of mixer stage is generally high.
Hence, a large signal is required to maintain an acceptable SNR.
Thus, weak signals are first amplified by an RF amplifier and then applied to mixer stage.

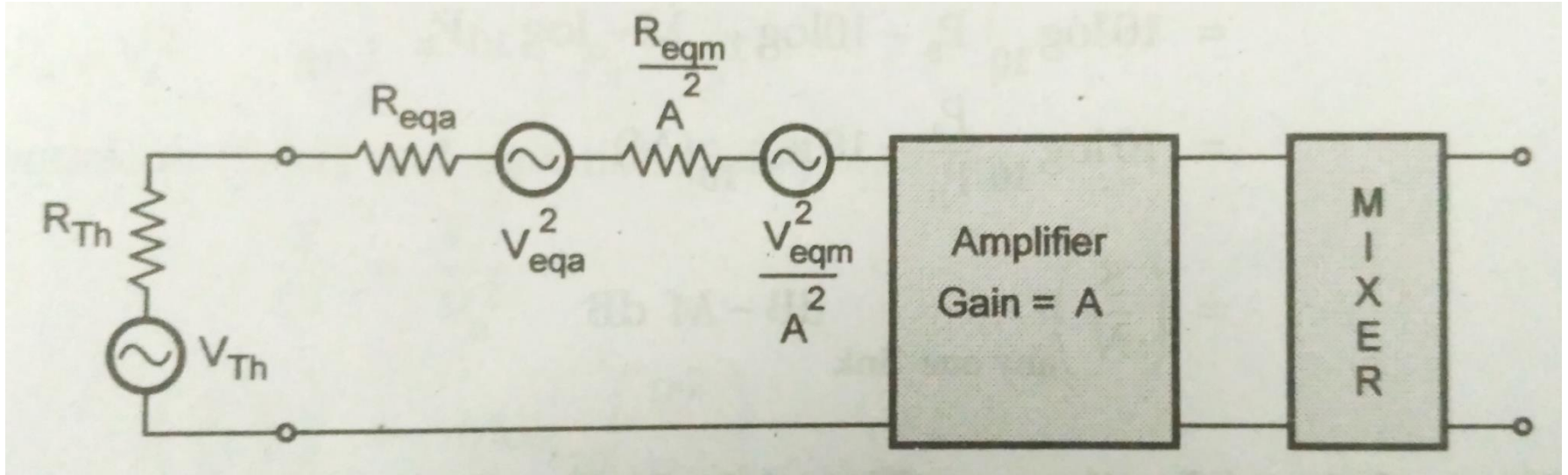RF amplr should have low noise (R_{eqa}) as compared to mixer stage (R_{eqm}).
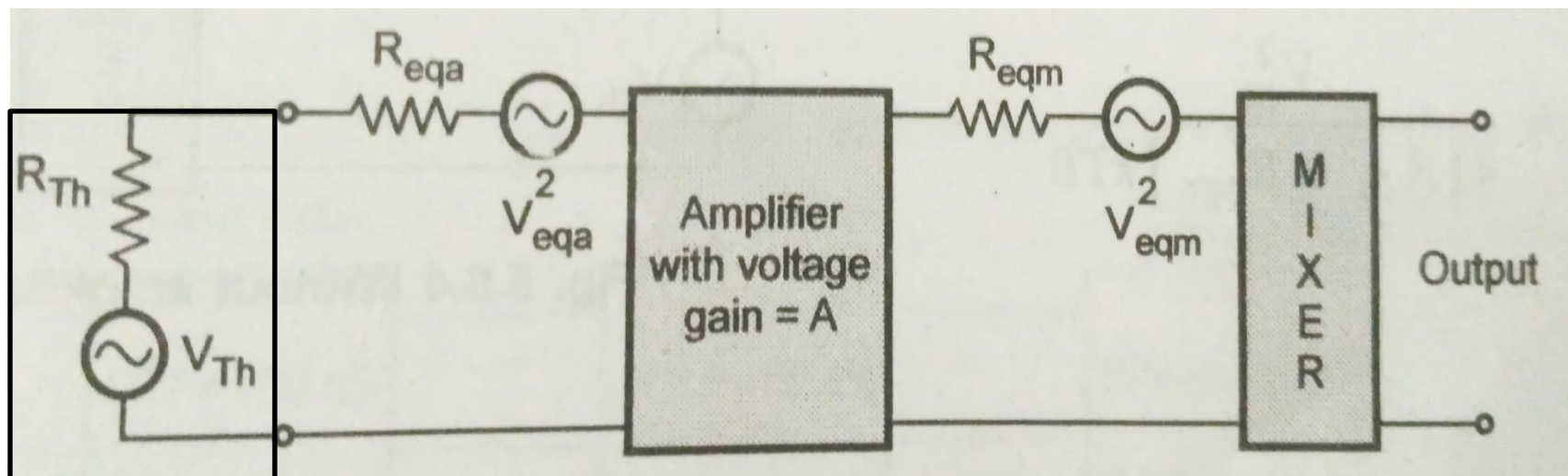
Signal ckt.

R_{eqa}, R_{eqm} : Equivalent noise resistance of amplifier stage and mixer stage respe.

V_{eqa}, V_{eqm} : Mean sq. noise voltages of amplifier stage and mixer stage respe.

V_{eqm}^2, R_{eqm} at the o/p of amplr can be replaced with $\frac{V_{eqm}^2}{A^2}$, $\frac{R_{eqm}}{A^2}$ at the input of amplr.

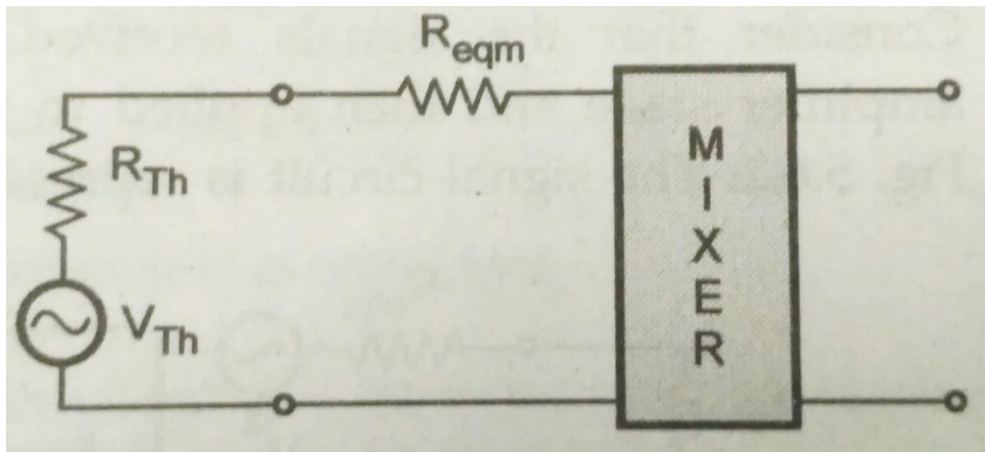$$V_n^2 = V_{eqa}^2 + \frac{V_{eqm}^2}{A^2} = 4\left(R_{eqa} + \frac{R_{eqm}}{A^2}\right) KTB_n$$

The SNR is,

$$SNR = \frac{V_{signal}^2}{V_{noise}^2} = \frac{V_{Th}^2}{4\left(R_{Th} + Reqa + \frac{R_{eqm}}{A^2}\right)KTB_n}$$

Effect of amplification on signal to noise ratio contd ….

If the signal were applied w/o amplification,
Then,

$$SNR = \frac{V_{signal}^2}{V_{noise}^2} = \frac{V_{Th}^2}{4(R_{Th} + R_{eqm})KTB_n}$$

Hence, SNR improves with the use of amplification.

Numericals: v

Noise factor:

Noise factor F of an amplifier or any other network is defined as the ratio of available SNR at the input to available SNR at the output.

i.e. Noise factor F = $\dfrac{(Psi/Pni)}{(Pso/Pno)}$

SNR at the i/p is always greater than SNR at the o/p. Hence, F is always greater than unity.

We know,

$$\text{Noise factor F} = \frac{(Psi/Pni)}{(Pso/Pno)} = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}}$$

$$= \frac{P_{no}}{P_{ni}} \times \frac{P_{si}}{P_{so}}$$

$$= \frac{P_{no}}{P_{ni}} \times \frac{1}{G} \qquad \{ \text{where, } G = \frac{P_{so}}{P_{si}} \}$$

$$P_{no} = P_{ni}FG$$

But, $P_{ni} = KTB_n$, **$P_{no} = KTB_nFG$**

Also, $P_{ni\_total} = P_{no}/G$, $P_{ni\_total} = FKTB_n$

Noise power of amplr = total P_{ni} - P_n due to src.

$$= FKTB_n - KTB_n$$

$P_{na} = (F-1)KTB_n$

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Noise figure:

Noise factor expressed in decibels, is called as Noise figure.

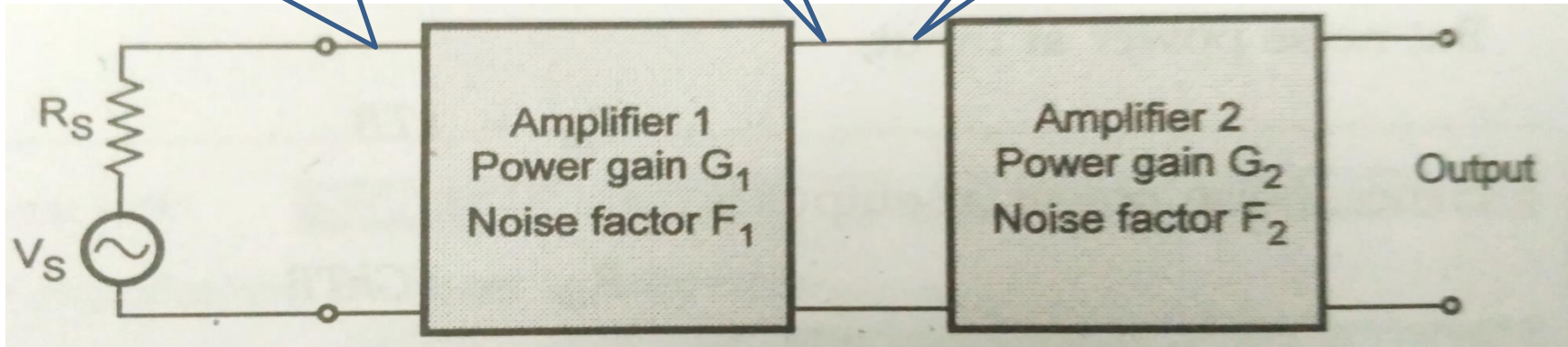 i.e. **Noise figure = (Noise factor)$_{dB}$**

Noise factor of amplrs in cascade:

$KTB_n + (F_1-1)KTB_n$
$= KTB_n + F_1 KTB_n - KTB_n$
$= F_1 KTB_n$

$F_1 G_1 KTBn$

$F_1 G_1 KTB_n + (F_2-1)KTB_n$

R_S

V_S

Amplifier 1
Power gain G_1
Noise factor F_1

Amplifier 2
Power gain G_2
Noise factor F_2

Output

$P_{no} = G_2 \{F_1 G_1 KTB_n + (F_2-1)KTB_n\}$
$\quad = \{F_1 G_1 G_2 KTB_n + (F_2-1) G_2 KTB_n\}$
$\quad = F_1 G KTB_n + (F_2-1)G_2 KTB_n$ ....{ where, $G = G_1 G_2$}

We know, $\mathbf{P_{no}= FGP_{ni}}= FGKTB_n$

$$\mathbf{F} = \frac{\mathbf{P_{no}}}{\mathbf{G\ Pni}} = \frac{P_{no}}{GKTB_n} = \frac{F_1GKTB_n+(F_2-1)G_2KTB_n}{GKTB_n} \quad ....\{ \text{where, } G= G_1G_2\}$$

$$F = F_1 + \frac{(F_2-1)}{G_1}$$

Similarly,

$$\mathbf{F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1G_2} + ...} \qquad \text{Friis's formula}$$

Noise factor in terms of R$_{eqa}$:
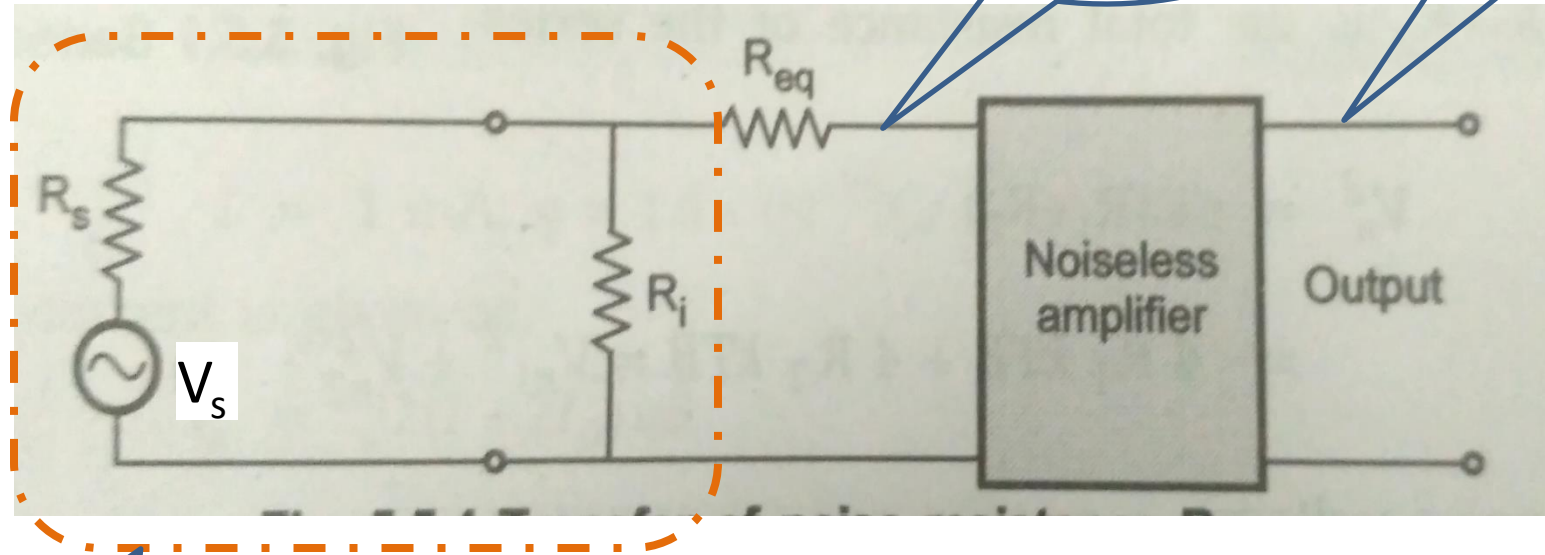
$(\frac{S}{N})'_{output}$

$(\frac{S}{N})_{output}$

$(\frac{S}{N})_{input}$

Noise factor $= \dfrac{\left(\frac{S}{N}\right)_{input}}{\left(\frac{S}{N}\right)_{output}}$

$= \dfrac{V_{Th}{}^2/4KTBnRTh}{V_{Th}{}^2/\ 4KTBn(RTh+Reqa)}$

where, $R_{Th} = R_s \| R_i$

Noise factor = 1+ $\dfrac{R_{eqa}}{R_{Th}}$

Numericals: viii, ix

Noise temperature:

Eq. noise temp. is defined as a temp. at which a noisy resistor connected at input of noiseless version of system produces the same noise pwr at the o/p of system as that produced by all src's of noise in the actual system.

- The noise power at the i/p of amplr is,

$P_{na} = KT_eB$

$T_e = \dfrac{P_{na}}{KB} = \dfrac{(F-1)KTB}{KB} = (F-1)T$

- Eq. noise temp. T_e is just an alternative way of representing noise factor.

Note: Eq. noise temp. is better measure for low noise devices i.e. low noise amplrs in satellite receiving systems, while noise factor is better measure for the main receiving systems.

Noise temp. of cascaded stages:

From Friis's formula,

$$F = F_1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \text{........}$$ **Friis's formula**

Subtract 1 from both sides,

$$F-1 = F_1 - 1 + \frac{(F_2-1)}{G_1} + \frac{(F_3-1)}{G_1 G_2} + \text{........}$$

$$(F-1)T = (F_1 - 1)T + \frac{(F_2-1)T}{G_1} + \frac{(F_3-1)T}{G_1 G_2} + \text{........}$$

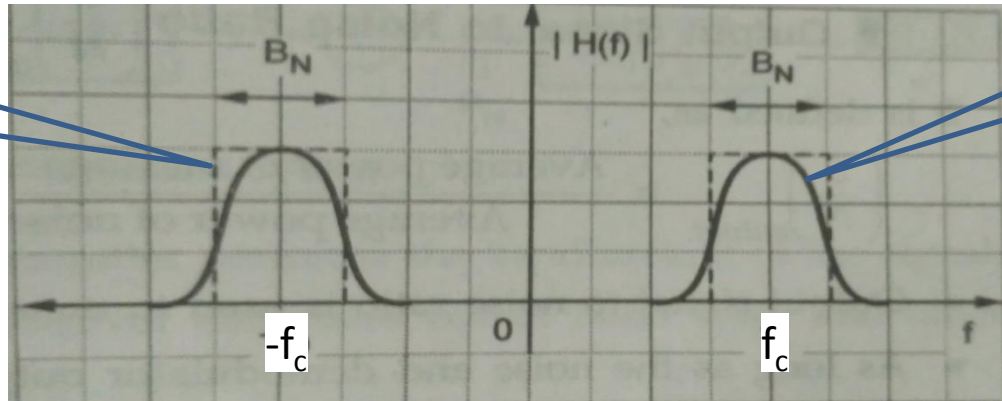$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \text{........}$$

Numericals: x,xi,xii

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Noise bandwidth:

Noise bandwidth B_N is defined as the bandwidth of an ideal filter which passes the same noise power as that of real filter.

$S_n(f) = N_0/2$; where $N_0 = KT_e$, K=Boltzmann's constant, T_e= Eq. noise temp. of system

For Arbitrary filter,

$$P_N = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)|^2 df = N_o \int_0^{+\infty} |H(f)|^2 df \quad \text{---------------------------(i)}$$

For ideal filter,

$$P_N = \frac{N_0}{2} 2B_N |H(f_c)|^2 = N_0 B_N |H(f_c)|^2 \quad \text{----------------------------------(ii)}$$

From (i) and (ii),

$$B_N = \frac{\int_0^{+\infty} |H(f)|^2 df}{|H(fc)|^2} \quad \text{------------------------------------(iii)}$$

Behavior of systems in presence of noise:

We will compare different analog modulation techniques in presence of noise.
Noise at Rx input is very much important because at Rx input, signal level and noise level are more or less equal.

Behavior of systems in presence of noise:

Tx noise + Rx noise + channel noise
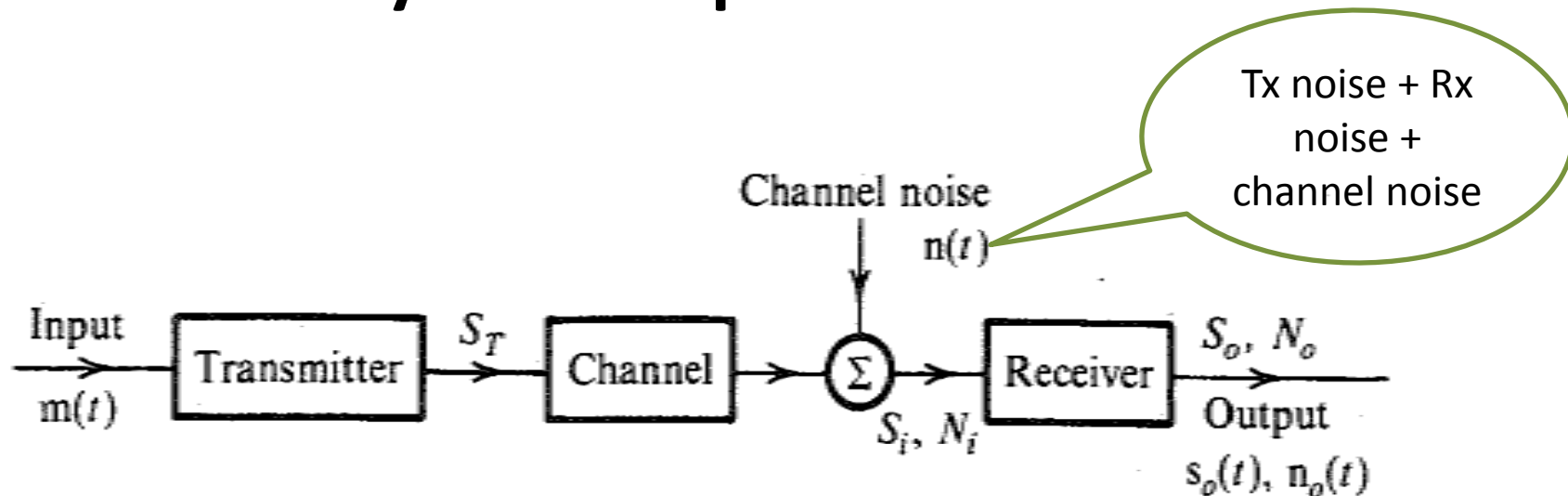
S_T: Transmitted power
S_i, S_o : Signal pwr at Rx i/p and o/p respe.
N_i, N_o : Noise pwr at Rx i/p and o/p respe.
$\dfrac{S_o}{N_o}$ = Figure of merit for any communication system.

e.g. Voice signal: (S/N)$_{dB}$*(typ. Value)*   Remark

	(S/N)$_{dB}$ *(typ. Value)*	Remark
	5-10dB	Barely intelligible
	25-35dB	Telephone quality
	45-55dB	Broadcast quality
Television	45-55dB	Broadcast quality

10dB: Signal is 10 times stronger than noise.
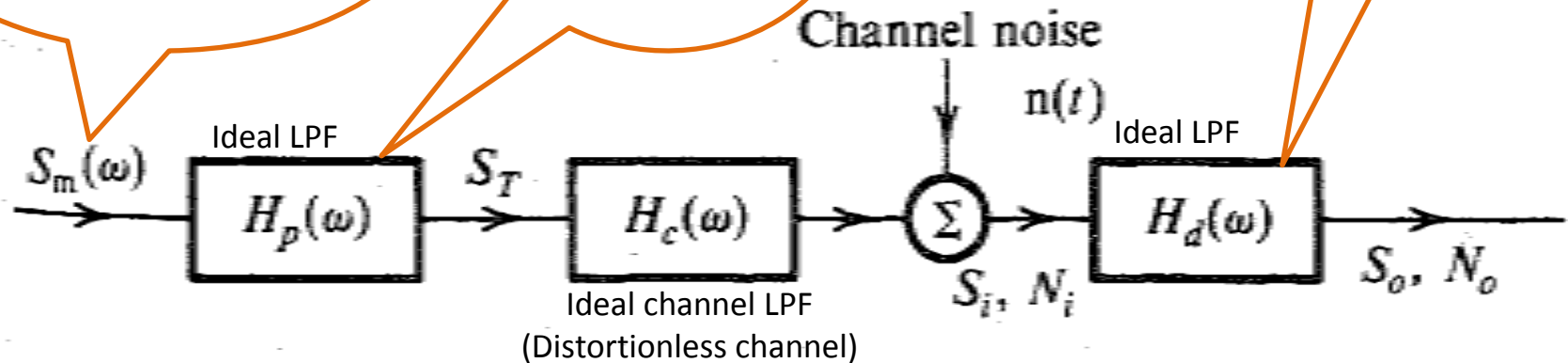50dB: Signal is 1 lac times stronger than noise.

1. Baseband systems: (Xmission w/o modulation)

(pair of wires/ coaxial cables connected bet^n home and exchange)

m(t) Necessarily random, zero mean random process, band limited to B Hz

Limits the spectrum to given bandwidth 'B'

Eliminates the out of band spectrum as well as noise

Ideal LPF

Ideal LPF

Channel noise

$n(t)$

$S_m(\omega)$ → $H_p(\omega)$ → S_T → $H_c(\omega)$ → Σ → $H_d(\omega)$ → S_o, N_o

S_i, N_i

Ideal channel LPF
(Distortionless channel)

We have considered everything ideal except n(t), as we want to study effect of noise.
Hence, signal power at Rx o/p = signal power at Rx i/p

i.e. **$S_o(t) = S_i(t)$**

Noise at Rx i/p, i.e. **$n_i(t) = \infty$** , because n(t) is assumed to be white noise.
When this n(t) is passed thr' $H_d(w)$, i.e. ideal LPF, it will become,

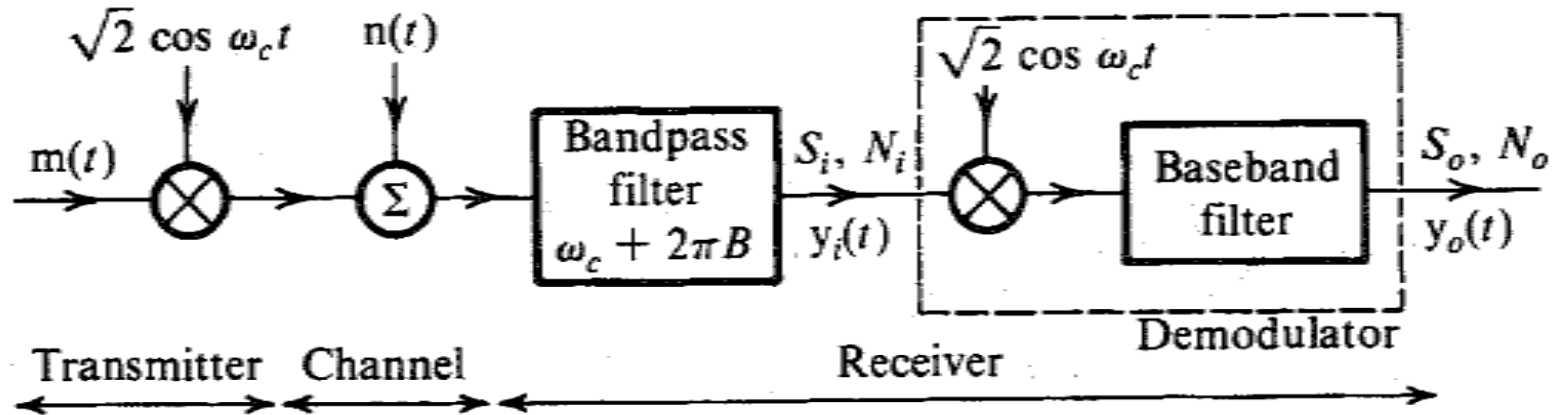$$\mathbf{N_o = n_o(t) = \int_{-\infty}^{+\infty} S_{no}(f) \cdot |H_d(f)|^2 \cdot df}$$

$$= \int_{-B}^{+B} \frac{N}{2} \cdot (1) \cdot df = 2\int_{0}^{+B} \frac{N}{2} \cdot (1) \cdot df = NB$$

$$\left(\frac{\mathbf{S_o}}{\mathbf{N_o}}\right) = \frac{\mathbf{S_i}}{NB} = \boldsymbol{\gamma}$$

The power or the mean square value of m(t) is $\overline{m}^2$,

$$\overline{m}^2 = 2 \int_0^B S_m(f) \, df$$

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2. DSBSC systems:

$S_i = E\{\sqrt{2}\ m(t)\cos w_c t\ \}^2$

$\quad = E\{2\ m^2(t)\cos^2 w_c t\}$

$\quad = E\{\ 2\ m^2(t)\left[\dfrac{(1+\cos 2wct)}{2}\right]$

$\quad = E\{m^2(t) + m^2(t)\cos 2wct\ \}$

$S_i = m^2(t)$

$\quad = \overline{m^2}$

In phase component

Quadrature component
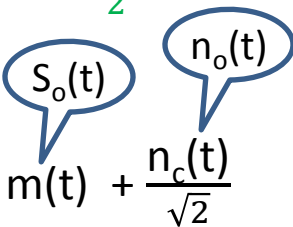
$y_i(t) = S_i(t) + n_i(t)$

$\quad = \sqrt{2}\ m(t)\cos w_c t + n_c(t)\cos w_c t + n_s(t)\sin w_c t$

$y_o(t) = (\sqrt{2}\cos w_c t)\, y_i(t)$

$\quad = (\sqrt{2}\cos w_c t)\,\sqrt{2}\,m(t)\cos w_c t + (\sqrt{2}\cos w_c t)(n_c(t)\cos w_c t) + (\sqrt{2}\cos w_c t)(n_s(t)\sin w_c t)$

$\quad = 2\,m(t)\cos^2 w_c t + \sqrt{2}\,n_c(t)\cos^2 w_c t + \sqrt{2}\,n_s(t)\cos w_c t\,\sin w_c t$

Solving above with substitution and passing through LPF of bandwidth 'B' Hz,

$$\cos^2 w_c t = \frac{(1+\cos 2w_c t)}{2} \qquad \& \qquad \sin w_c t\,\cos w_c t = \frac{\sin 2w_c t}{2}$$

$S_o(t)$ $n_o(t)$

$$y_o(t) = m(t) + \frac{n_c(t)}{\sqrt{2}}$$
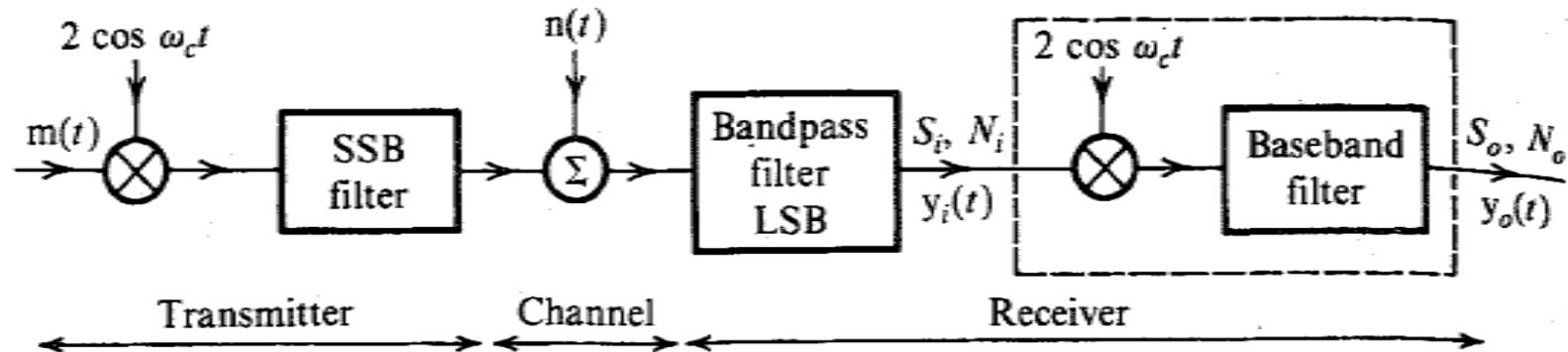
Hence,

$$S_o = \overline{m^2}$$

$$N_o = \left(\frac{n_c(t)}{\sqrt{2}}\right)^2$$

$$\quad = \frac{1}{2}\frac{N}{2}\,4B$$

$$\quad = NB$$

$$\frac{S_o}{N_o} = \frac{\overline{m^2}}{NB} = \frac{S_i}{NB} = \gamma$$

No difference between baseband and DSBSC systems for SNR.

3. SSBSC systems:

2 cos $\omega_c t$ n(t) 2 cos $\omega_c t$

m(t) → ⊗ → SSB filter → ⊗(Σ) → Bandpass filter LSB | S_i, N_i $y_i(t)$ → ⊗ → Baseband filter | S_o, N_o $y_o(t)$

Transmitter ← → Channel ← → Receiver

SSB is generated by multiplying 'm(t)' and '2 cos $w_c t$' & then
Passing it thr' SSB filter.

$$\varphi_{SSB}(t) = m(t) \cos w_c t \pm m_h(t) \sin w_c t$$

Consider, LSB SSB filter,

$$\varphi_{SSB}(t) = m(t) \cos w_c t + m_h(t) \sin w_c t$$

Power of '2m(t)cos $w_c t$' is,

$$= E\{2m(t)\cos w_c t \}^2$$
$$= 4 \, m^2(t) \cos^2 w_c t$$
$$= 4 \, m^2(t)\left[\frac{(1+\cos 2wct)}{2} \right]$$
$$= 2 \, m^2(t) + 2 \, m^2(t) \cos 2wct$$

If passed thr' DSB, power= $2\overline{m^2}$.

As passed thr' SSB filter, power= $\overline{m^2}$ i.e. $S_i = \overline{m^2}$

$y_i(t) = S_i(t) + n_i(t)$

$\quad = [m(t) \cos w_c t + m_h(t) \sin w_c t] + [n_c(t) \cos w_c t + n_s(t) \sin w_c t]$
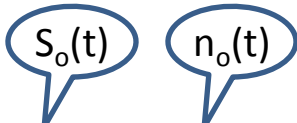
$y_o(t) = 2\cos w_c t \cdot y_i(t)$

$\quad = 2\cos w_c t \left\{ [m(t) \cos w_c t + m_h(t) \sin w_c t] + [n_c(t) \cos w_c t + n_s(t) \sin w_c t] \right\}$

$\quad = 2\, m(t)\cos^2 w_c t + 2\, m_h(t) \sin w_c t \cos w_c t + 2\, n_c(t) \cos^2 w_c t + 2 n_s(t) \sin w_c t \cos w_c t$

$\quad = 2m(t) \left[\dfrac{(1+\cos 2wct)}{2} \right] + 2\, n_c(t) \left[\dfrac{(1+\cos 2wct)}{2} \right]$

Passed thr' Baseband LPF,

$y_o(t) = \dfrac{2m(t)}{2} + \dfrac{2nc(t)}{2}$

$\overparen{S_o(t)} \quad \overparen{n_o(t)}$

$y_o(t) = m(t) + n_c(t)$

$S_o = \overline{m^2}$

$N_o = \overline{(n_c(t))^2}$

$\quad = \dfrac{N}{2}\, 2B$

$\quad = NB$

$\dfrac{So}{No} = \dfrac{\overline{m^2}}{NB} = \dfrac{S_i}{NB} = \gamma$

i.e. *No difference between baseband, DSBSC and SSBSC systems for SNR.*